The Chua-oscillator[1]

1 Introduction

Although unintended, it turned out that the first three issues of "Analog Computer Applications" are solely devoted to chaotic systems and their implementation on analog computers. This third issue deals with the so-called Chua-*oscillator*. This circuit was discovered in 1983 by Leon O. Chua[2] and is a classic example of an electronic circuit exhibiting chaotic behavior. This oscillator generates a rather unique and beautiful attractor (the *Double Scroll attractor*). It is based on a (rather hypothetical) nonlinear device called Chua-*diode* which can be implemented by means of a small operational amplifier circuit.

The mathematical description of this particular oscillator is based on three coupled differential equations of the form

$$\dot{x} = c_1(y - x - f(x)) \tag{1}$$
$$\dot{y} = c_2(x - y + z) \tag{2}$$
$$\dot{z} = -c_3 y, \tag{3}$$

where $f(x)$ describes the behavior of the Chua-diode and is traditionally defined as

$$f(x) = m_1 x + \frac{m_0 - m_1}{2} \left(|x + 1| - |x - 1| \right). \tag{4}$$

The standard values for the parameters are

$$c_1 = 15.6$$
$$c_2 = 1$$
$$c_3 = 28$$
$$m_0 = -1.143$$
$$m_1 = -0.714$$

2 Scaling

To implement this system of coupled differential equations on a traditional analog computer, all of its terms must be scaled so that

- no computing element will be driven into overload, so no term may exceed ± 1 machine unit and

- every computing element should be driven over as much of its voltage range as possible to minimize the influence of component errors etc.

This task is not trivial due to the nonlinear elements in these coupled differential equations. By computing example solutions on a digital computer – a technique that was already used in the 1960s – the ranges of the various terms was determined which were then used for guidance during the process of scaling.

The above equations were split into individual terms resulting in the following set of scaled equations. (1) is replaced by

$$x_0 = 0.1 \text{ (initial condition for the integration)} \tag{5}$$
$$x_1 = -10(x + f(x)) \tag{6}$$
$$x_2 = y + \frac{1}{2}x_1 \tag{7}$$
$$x = 3.12 \int x_2 \, \mathrm{d}t + x_0. \tag{8}$$

(2) is transformed into

$$y_1 = z - \frac{1}{8}y \tag{9}$$
$$y_2 = 1.25x + 2y_1 \tag{10}$$
$$y = 4 \int y_2 \, \mathrm{d}t, \tag{11}$$

while (3) becomes

$$z = -\int 3.5y \, \mathrm{d}t. \tag{12}$$

The central element, the Chua-diode is a bit nasty to implement as the absolute value function requires at least two diodes and an open amplifier for a proper implementation.[3] $f(x)$ is broken down and scaled into

---

[3]All in all four external diodes are required to implement the two absolute value functions.
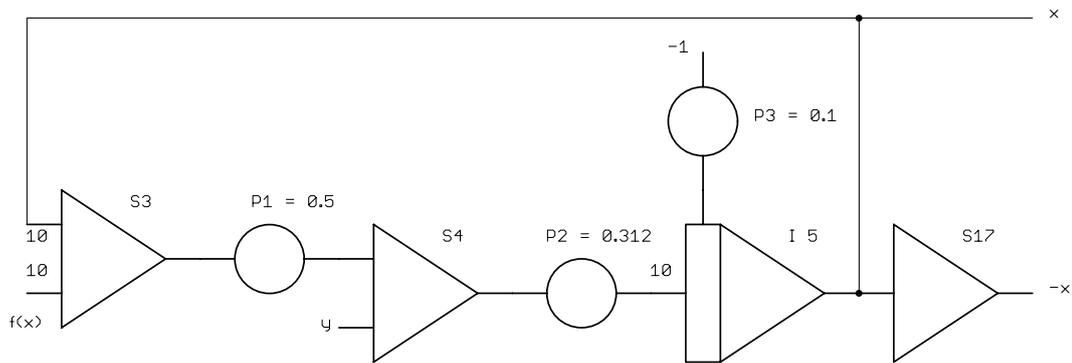
Figure 1: Partial computer setup for equations (5, 6, 7, 8)

$$f_1 = |0.7143x + 0.2857| \tag{13}$$
$$f_2 = |0.7143x - 0.2857| \tag{14}$$
$$f_3 = f_1 - f_2 \tag{15}$$
$$f(x) = -0.714x - 0.3003 f_3. \tag{16}$$

3 Computer setup

Deriving a computer setup from these equations is rather straight-forward.[4] The first set of equations, (5, 6, 7, 8), is implemented as shown in figure 1. The equations (9, 10, 11, 12) are implemented as shown in figure 2.

---

[4]The numbers of the computing elements shown in the following were originally geared towards a Telefunken RA 742 table-top analog computer and can be ignored if another machine is going to be used. Inputs denoted with a 10 are weighted inputs with a gain of 10 which is essential for the scaling to work.
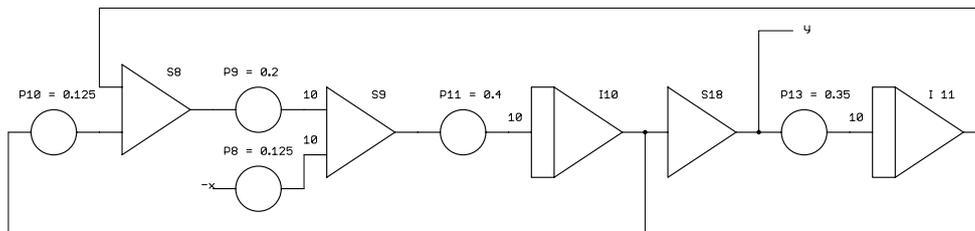
Figure 2: Partial computer setup for equations (9, 10, 11, 12)

The mechanization of the central function $f(x)$ is a bit awkward as two absolute value functions are required. The setup representing equations (13, 14, 15, 16) is shown in figure 3.

Figure 4 shows the program implemented on an early prototype of Analog Paradigm's Model-1 analog computer which features an absolute value function module, significantly simplifying the implementation of $f(x)$. In this case the summers S12, S13, S14, and S15 as well as the four free diodes can be replaced by two absolute value function modules.

4   Results

By changing the time-constant of the three integrators of this circuit, it is easily possible to either display the Double Scroll Attractor on an oscilloscope with x- and y-inputs (driven by x and y of the computer circuit) or to plot the attractor using a traditional pen plotter. Figure 5 shows a screen-shot from an oscilloscope's screen, generated on the `Analog Paradigm` Model-1 analog computer with the integrator time
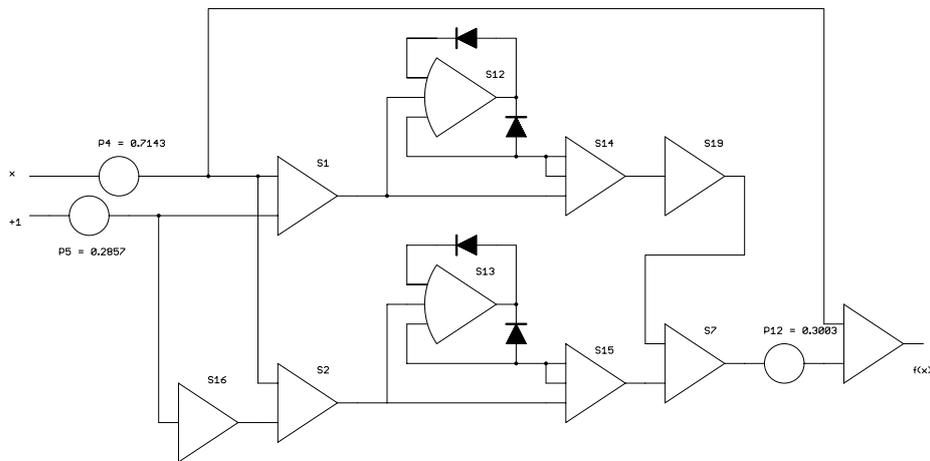
Figure 3: Partial computer setup for equations (13, 14, 15, 16)

constants set to 1000. Using slower time constants output on a plotter is, of course, also possible.
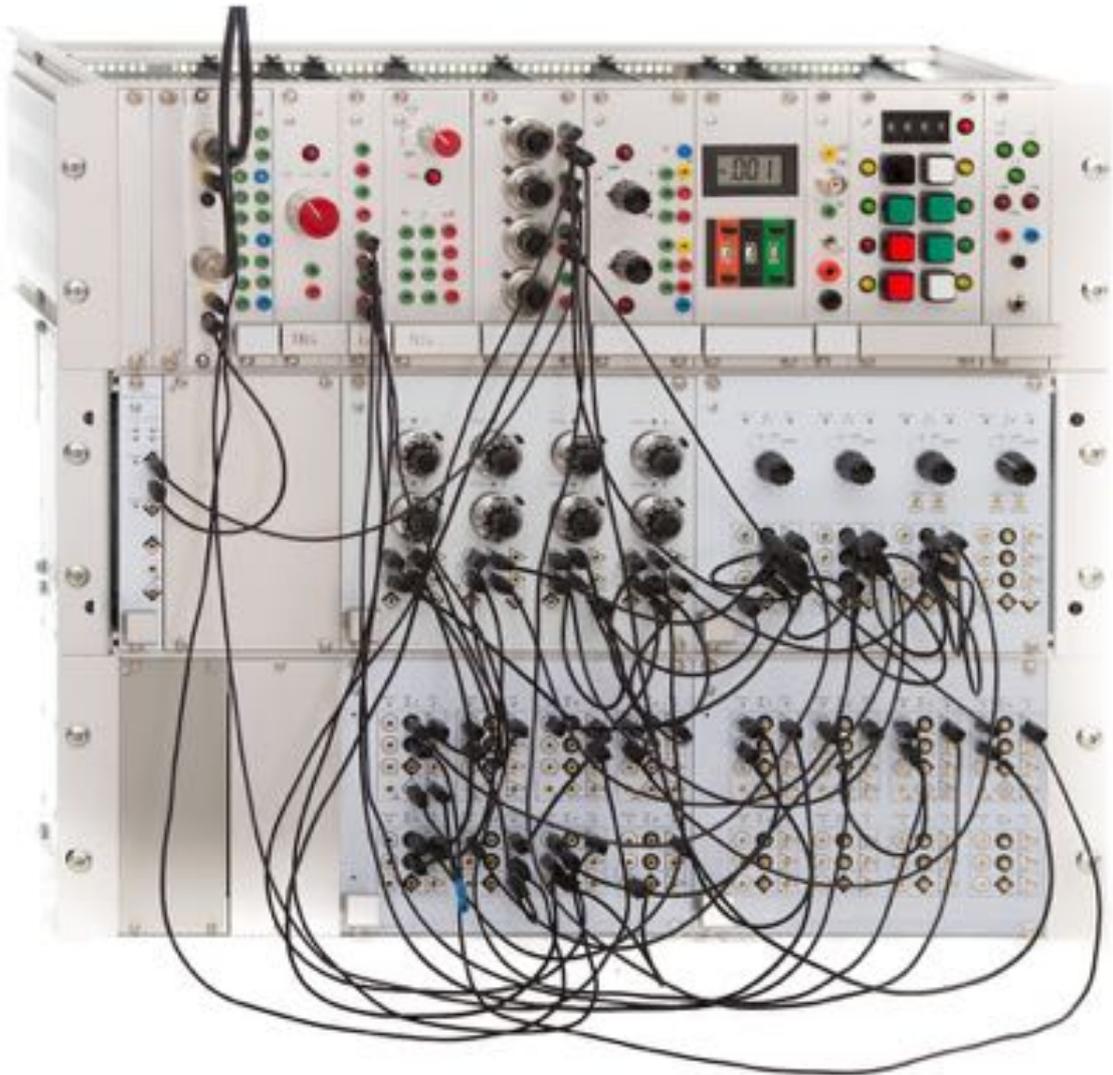
Figure 4: Setup of the Chua-oscillator on an early Analog Paradigm Model-1 analog computer
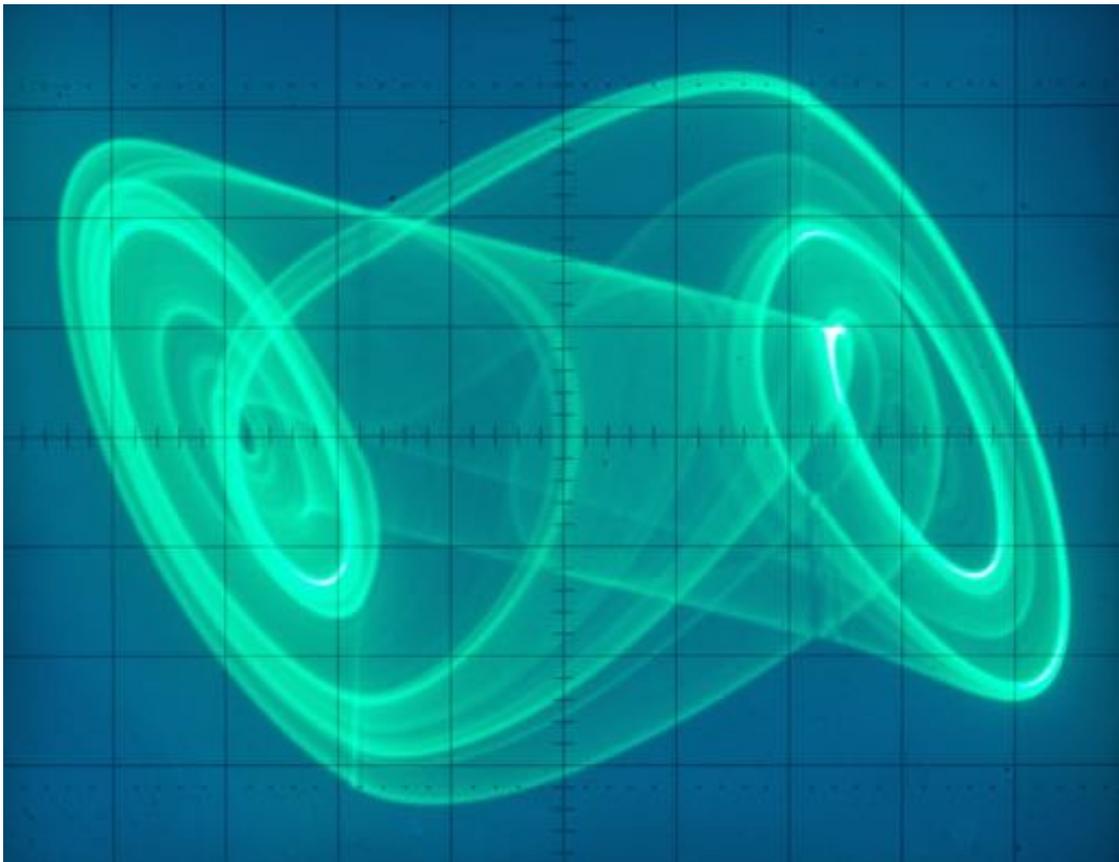
Figure 5: Phase space display of the Double Scroll attractor generated on an `Analog Paradigm` Model-1 analog computer