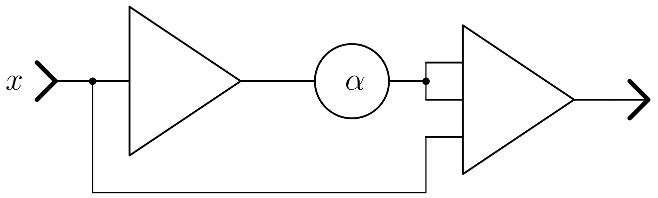

Figure 1: Generating coefficients within $[-1, 1]$

Displaying polynomials

An analog computer is a wonderful educational tool due to its unmatched high degree of interactivity. Running in repetitive mode of operation it is possible to immediately see the effects of changing parameters in a program on an oscilloscope screen.

This application note describes a simple analog computer program to display a polynomial of the form

$$p(x) = ax^3 + bx^2 + cx + d$$

with $a, b, c, d \in [-1, 1]$ which can be varied manually.[1]

Since typical coefficient potentiometers in an analog computer only allow values within the interval $[0, 1]$ to be set, a little trick is required to extend the coefficient interval to $[-1, 1]$. Figure 1 shows the basic idea: A summer is fed with some variable x as well as with $-2\alpha x$, $0 \le \alpha \le 1$. The factor 2 is implemented by feeding two inputs of the summer with the same value αx. Varying α between 0 and 1 will cause the output to linearly run from $-x$ to x.

Assuming that four terms x^3, x^2, x, and1 are available, this coefficient circuit can be employed four times yielding ax^3, bx^2, cx, and d. Summing these will then yield the desired polynomial.

[1]Certain combinations of a, b, c, d will, of course yield an overload, thus limiting the polynomial to values within the machine unit interval of $[-1, 1]$.

Figure 2 shows the overall circuit. Integrating once over a constant τ yields a linear ramp x. (In an ideal world, $\tau = 1$ would hold, but it is more convenient to have a way to tweak the steepness of the ramp so that the following higher powers of x have the required symmetrical shape.) Integrating over x with a factor of two yields (due to the implicit sign inversion every integrator and summer exhibits) $-x^2$. Integrating again, now with a factor of three, yields x^3.[2]

Using two summers with six inputs each, four of the coefficient circuits shown in figure 1 are implemented. Since THE ANALOG THING only has summers with four inputs with weight 1, the summing junctions (SJ) of two of these summers have to be connected to SJ of one of the free resistor arrays in the lower right corner of the patch panel each, thus expanding the number of inputs of these two summers. Feeding the outputs of these two summers into a third summer finally yields the desired polynomial.

Figure 3 shows the three individual terms x, x^2, and x^3 generated by this circuit. By changing the coefficients a, b, c, and d a polynomial $p(x) = ax^3 + bx^2 + cx + d$ can be displayed.

The overall setup on THE ANALOG THING is shown in figure 4. The circuit requires three integrators, three summers, both of the free resistor networks, three inverters, and five coefficient potentiometers. To display this function on an oscilloscope, x and $p(x)$ are fed to both inputs of an oscilloscope set to x/y-mode.[3]

[2]Although the last two of these integrators could be replaced by two multipliers, fed with x and $-x$, this approach of generating higher powers of x by successive integration is a useful technique for saving expensive multipliers.

[3]Ideally, the TRIG output of THE ANALOG THING is connected to the z modulation input of the oscilloscope to suppress the beam return which can be seen in the screen shots here.

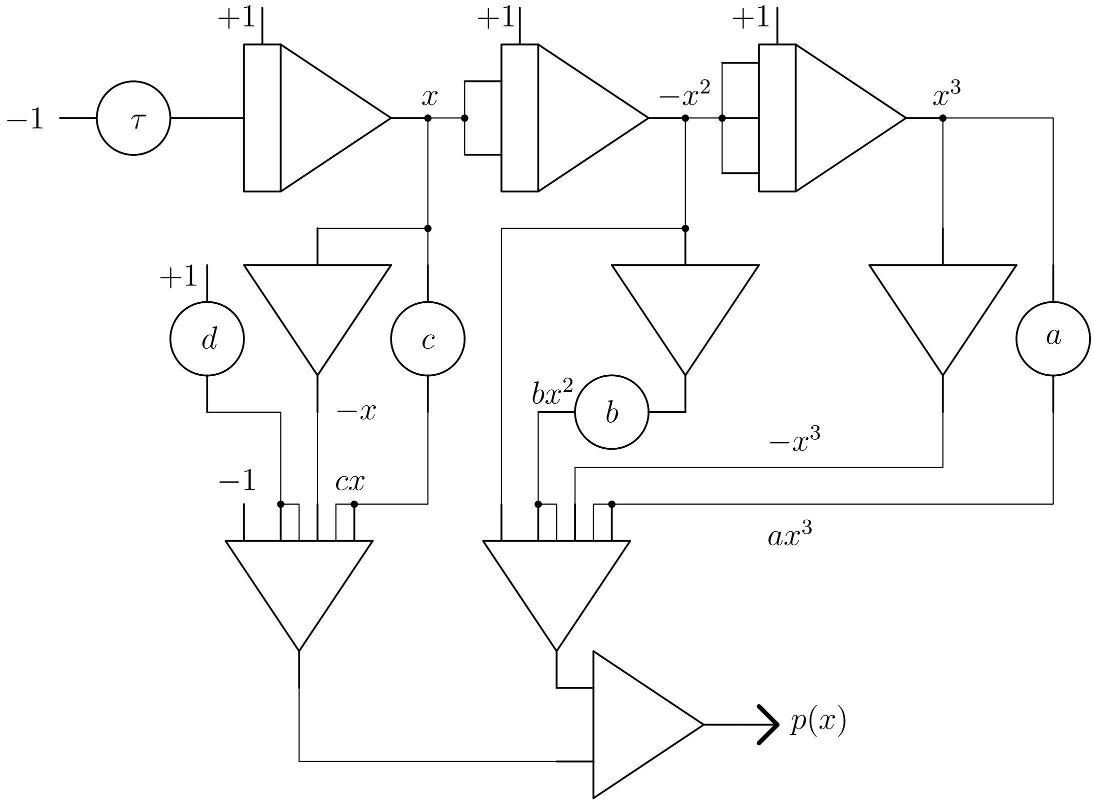

Figure 2: Generating $p(x) = ax^3 + bx^2 + cx + d$ with $a, b, c, d \in [-1, 1]$

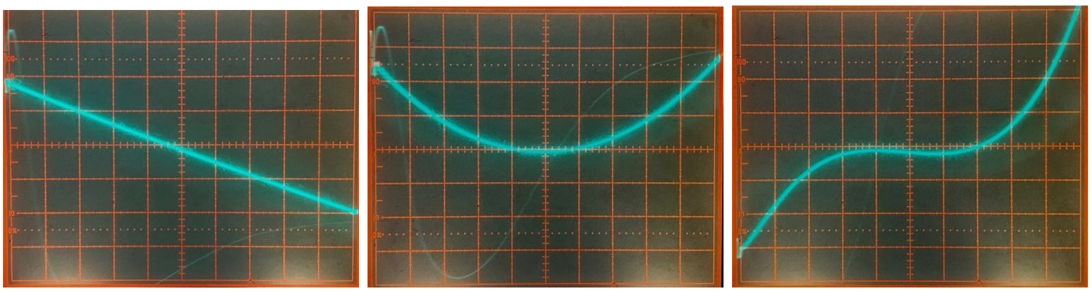

Figure 3: Basic terms x, x^2, and x^3

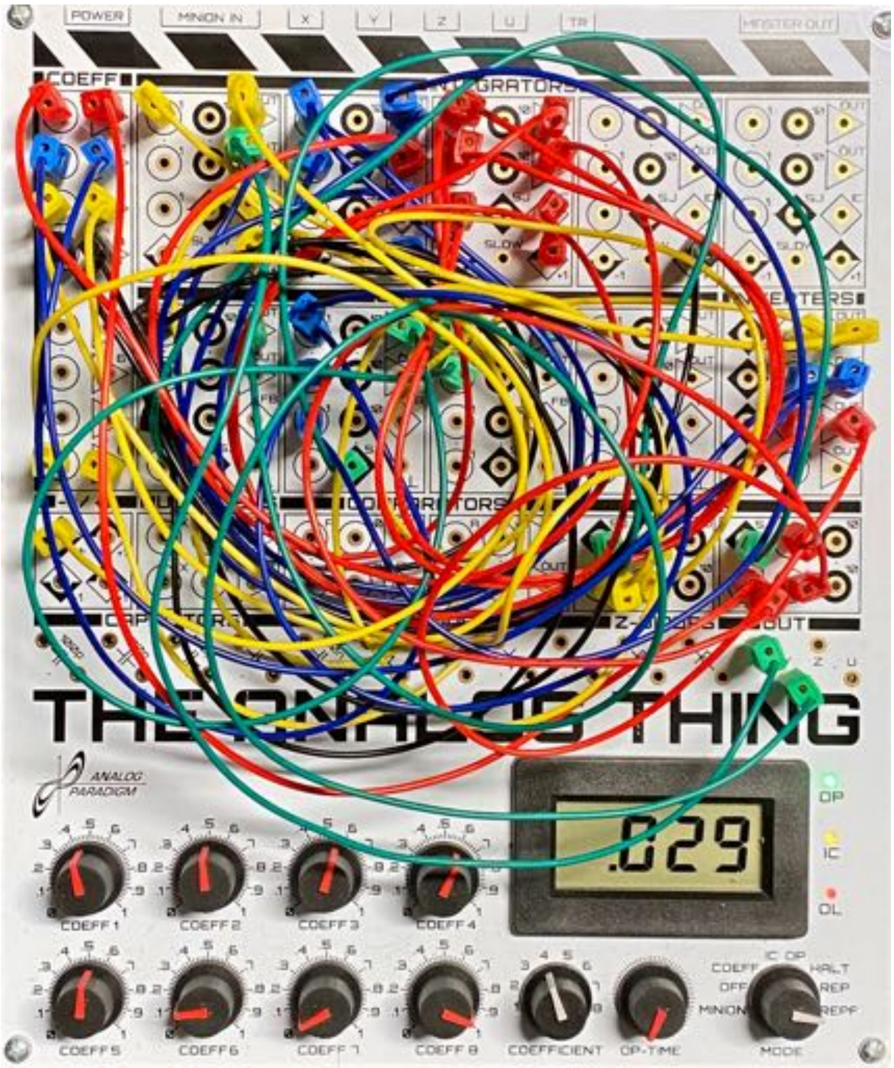

Figure 4: Setup of the polynomial circuit on THE ANALOG THING