



Figure 1: Scaled analog computer setup for the $\operatorname{HINDMARSH-ROSE}$ model

Experiments with Hindmarsh-Rose neurons

The HINDMARSH-ROSE model of neural bursting was already topic of application note $#28^1$. This application note extends such neurons with excitatory and inhibitory inputs allowing to couple several neurons to obtain more complex behavior. The basis of this is the analog circuit described in application note #28, shown in figure 1. It is controlled by the input I_{ext} fed to the integrator in the upper left.

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¹See https://analogparadigm.com/downloads/alpaca_28.pdf.





Figure 2: Adding excitatory and inhibitory inputs to a neuron

Neurons typically feature excitatory and inhibitory inputs, which must be summed over in order to "trigger" them. This is done by the little circuit shown in figure 2. The output signals x and -x of a preceding neuron are used to generate exitatory and inhibitory signals for a subsequent neuron. Since the output x of a HINDMARSH-ROSE neuron as shown in figure 1 at rest is < 0 a diode is necessary to filter out only the positive going spikes for excitatory inputs. Inhibitory inputs are fed with -x and a diode with reverse orientation to filter out only the negative going spikes. Excitatory and inhibitory signals will be denoted by x_E and $-x_I$ (or simply + and -) in the following with E and I denoting suitable index sets. Each input has a synaptic weight w_E or w_I respectively.

The circuit shown only has one input of each type. Additional inputs require additional diodes connected to inputs of the integrator which basically sums over the input spikes it is presented with. The negative feedback γ makes sure that the output signal of the itegrator will decay according to an exponential function over time thus making sure that it will not trigger the following neuron indefinitely after enough excitatory input spikes have accumulated once.

In the following, a HINDMARSH-ROSE neuron with such an input stage will be represented by the symbol shown in figure 3. A simple circuit consisting of two such neurons is shown in figure 4. Neuron 0 is continuosly bursting due to the +1 at its sole excitatory input. It then triggers neuron 1 which will also yield a burst with a slight time delay (depending on the actual values for its synaptic weight w and γ . Figure 5 shows a typical result.

What happens when the output of neuron 1 is fed back to neuron 0 as an inhibitory signal as shown in figure 6? At first glance the result shown in figure 7 looks similar to figure 5

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Figure 3: Symbol for a $\operatorname{HINDMARSH-ROSE}$ neuron with an input stage as shown in figure 2





Figure 4: Neuron 0 is continuously bursting and triggers neuron $1 \label{eq:rescaled}$

Figure 5: Neuron 0 (blue) triggers neuron 1 (violet)

but the duration of the bursts is much shorter due to the inhibory signal from neuron 1 to neuron 0.

Using a single neuron it is also possible to construct a flip-flop as shown in figure 8. It is started with a short excitatory input of +1, holds its state with input 0, and is reset by a short inhibitory input of -1. Its behavior is shown in figure 9. The section with enlarged spike amplitudes on the left is due to the exitatory start input +1, the constant lower amplitudes represent the set-state. Resetting it with -1 quenches the spiking.

Figure 10 shows the setup used in the above examples.

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Figure 6: Neuron 0 is triggered by a constant +1 as excitatory input, triggering neuron 1 which in turn inhibits neuron 0



Figure 7: Result of the setup shown in figure 6



Figure 9: Behavior of a single-neuron flip-flop

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Figure 8: Simple flip-flop based on a $\rm HIND-MARSH-ROSE$ neuron with excitatory feedback





Figure 10: Setup used in these experiments