A four-wing attractor

This application note describes a rather beautiful "four-wing attractor", which is due to [Wang et al. 2009]. In this paper the authors analyse a general chaotic system of three coupled differental equations of the form

$$\begin{aligned}\dot{x} &= ax + cyz \\ \dot{y} &= bx + dy - xz \\ \dot{z} &= ez + fxy.\end{aligned}$$

A particularly beautiful attractor results from the parameter set $a = 0.2$, $b = -0.01$, $c = 1$, $d = -0.4$, $e = -1$, and $f = -1$. The resulting set of equations

$$\begin{aligned}\dot{x} &= 0.2x + yz \\ \dot{y} &= -0.01x - 0.4y - xz \\ \dot{z} &= -z - xy\end{aligned}$$

is easy to scale since a quick numeric experiment shows that x, y, and z are well within the interval $[-3, 3]$, so that scaling coefficients $\lambda_x = \lambda_y = \lambda_z = \frac{1}{3}$ may be used. The resulting scaled system of equations is then

$$\begin{aligned}\dot{x} &= 0.2x + 3yz \\ \dot{y} &= -0.01x - 0.4y - 3xz \\ \dot{z} &= -z - 3xy,\end{aligned}$$

which can be implemented directly on an analog computer.[1]

The resulting analog computer setup is shown in figure 1. Figures 2 and 3 show the behavior of this system in the y/z- and x/y-plane respectively.[2]

Happy analog computing! :-)

[1] Since it requires three products either two coupled THE ANALOG THINGs or an external multiplier such as the AD633 are required.

[2] It may be necessary to twiddle the parameter 0.01 slightly to get a nice result.

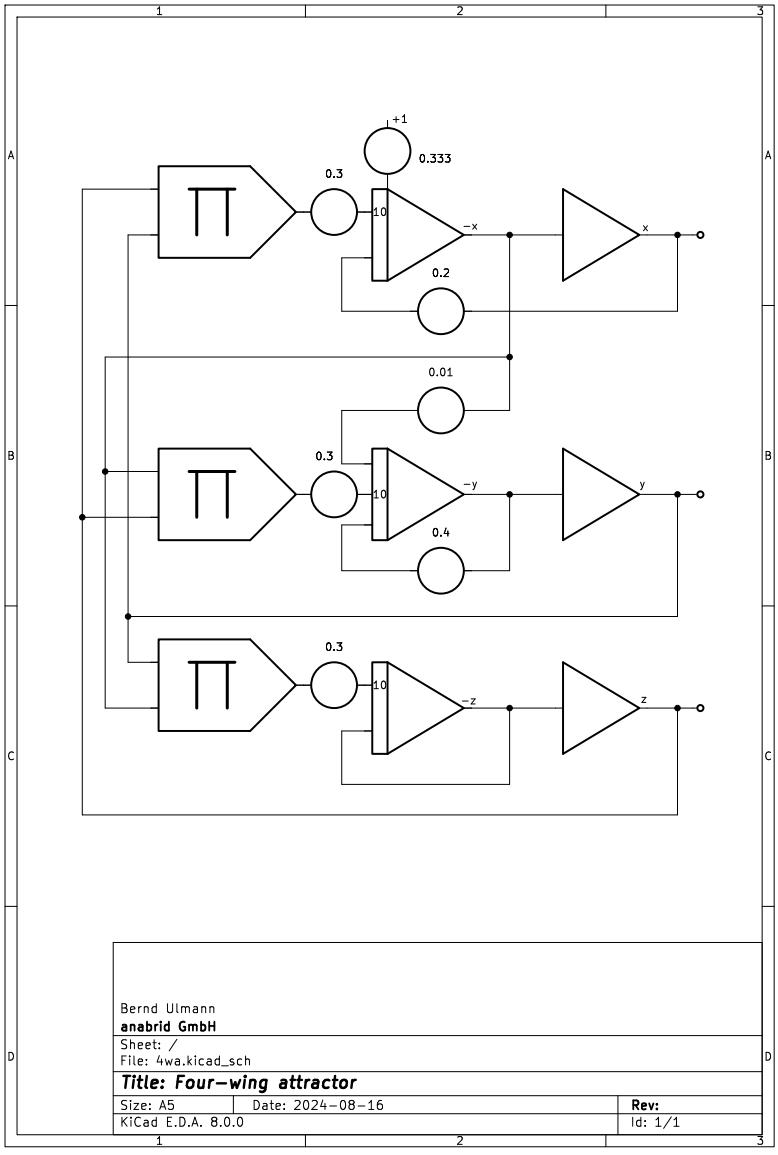

Figure 1: Analog computer setup for the three time scale system

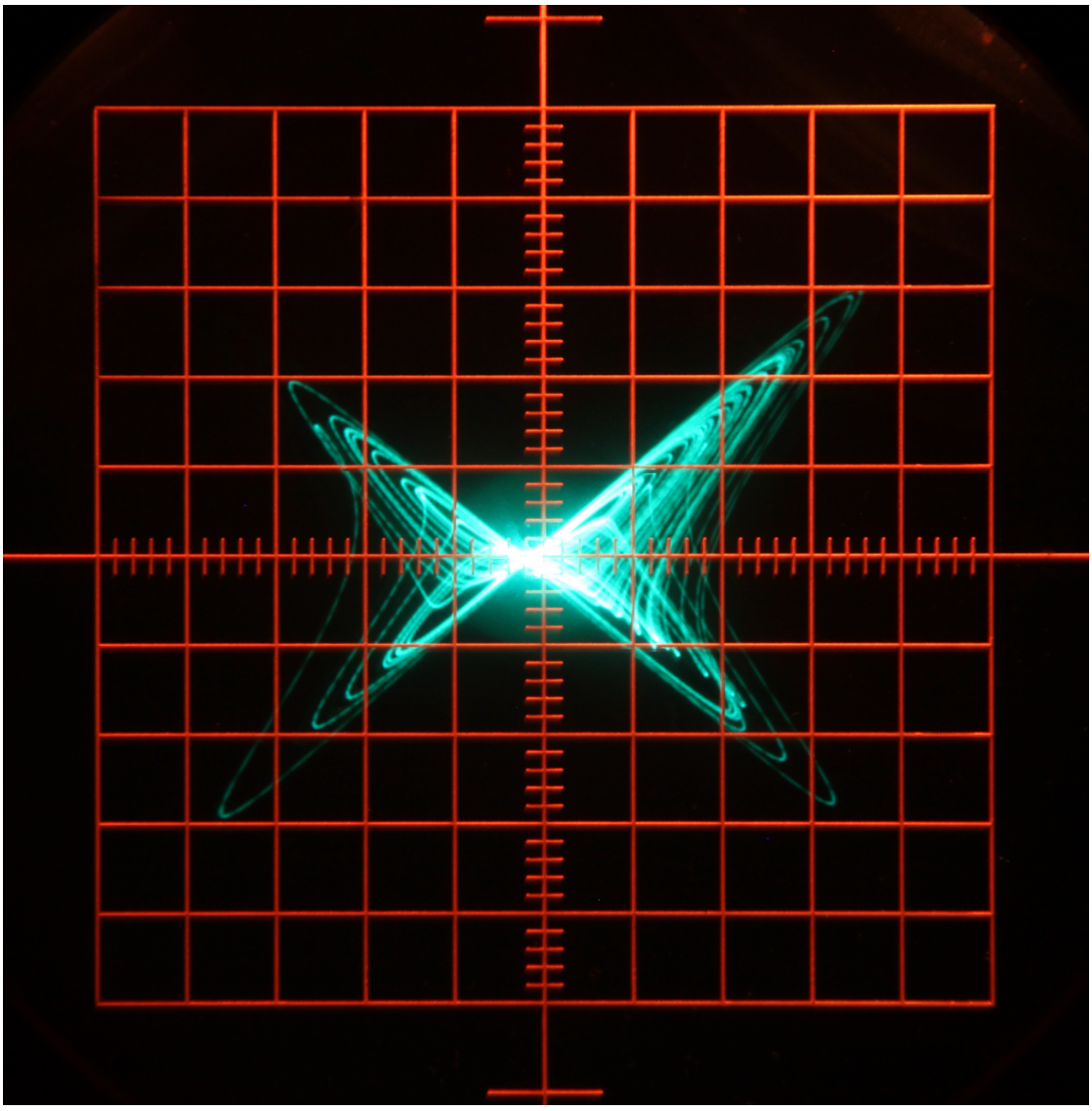

Figure 2: Four-wing attractor in the y/z-plane

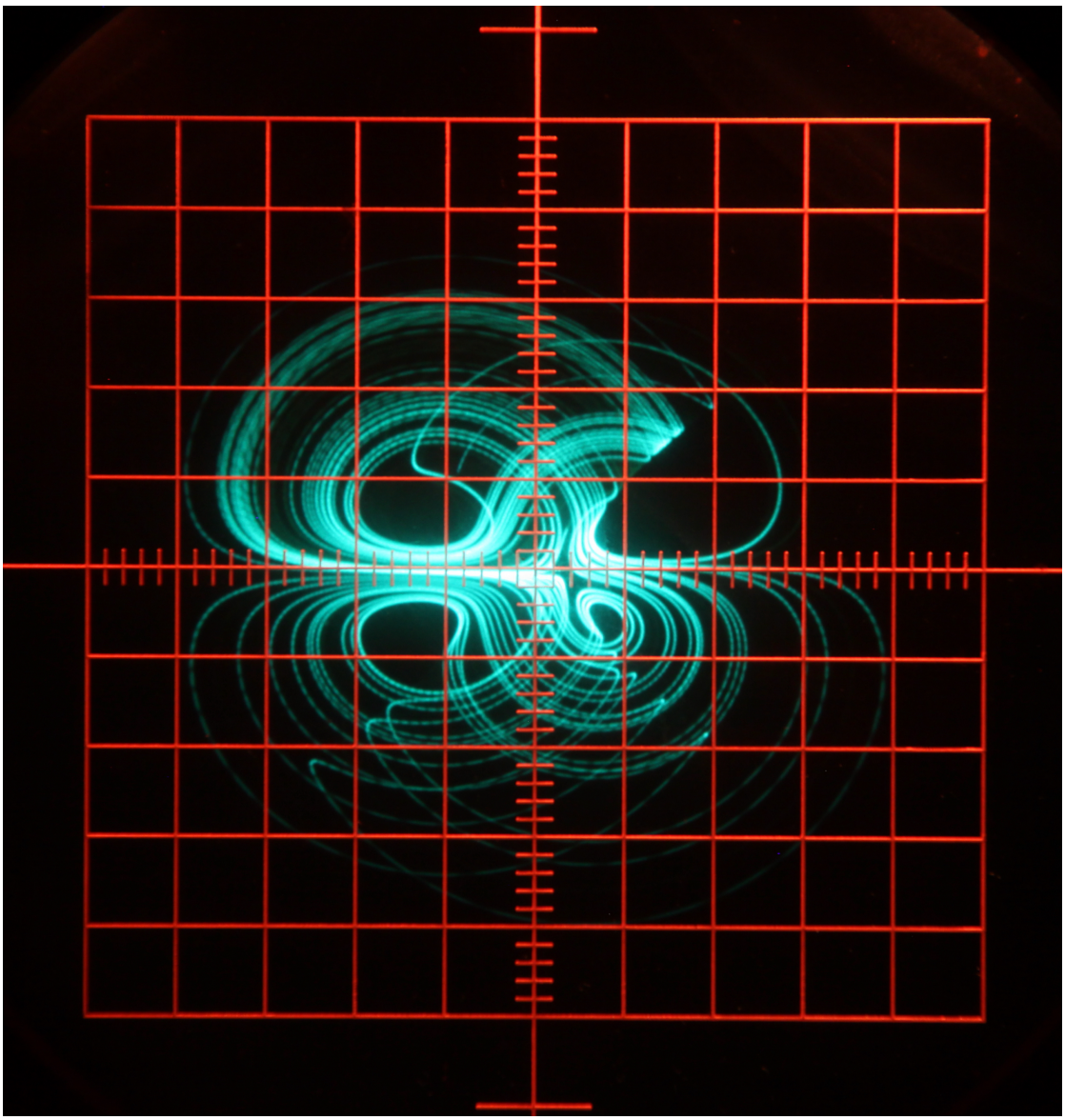

Figure 3: Four-wing attractor in the x/y-plane

References

[Wang et al. 2009] Zenghui Wang, Yanxia Sun, Jacobus van Wyk, Guoyuan Qi, Michael Antonie van Wyk, "A 3-D four-wing attractor and its analysis", in *Brazilian Journal of Physics*, vol. 39, no. 3, September, pp. 547–553