think hybrid

Analog Computer Applications

A simple memristor model

1 Introduction

In 1971, Leon O. Chua proposed a novel passive electronic circuit element[1] as the fourth basic circuit element complementing the resistor, inductor, and capacitor. Back then it was a purely theoretical concept, which only became a reality in recent years with several companies working in that field. Chua describes this circuit element as being "*characterized by a relationship between the charge* $q(t) \equiv \int_{-\infty}^{t} i(\tau)\,d\tau$ *and the flux-linkage* $\phi(t) \equiv \int_{-\infty}^{t} v(\tau)\,d\tau$". This element is called *memristor*, short for *memory resistor*. Figure 1 shows the inofficial symbol for a memristor.

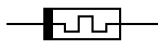

Figure 1: Memristor symbol (as suggested by Chua)

The first practical laboratory implementations of memristors became available beginning in 2007.[2] Memristors show big commercial potential in the near future as storage elements, as the basis for artificial neural networks (in the form of synaptic weights), etc.[3]

2 Model and results

The following memristor model is very simple and not too realistic. It models a device that is voltage controlled instead of current controlled and acts as a unidirectional component, i. e., it has a distinctive input and output connection, in contrast to a real memristor which acts like a resistor with some internal memory. The underlying idea of the model is to integrate

[1] See [Chua 1971].

[2] Currently, memristors can be bought from companies like Knowm (`https://knowm.org`) or Techifab (`https://techifab.com`) for experimental purposes.

[3] See [Schmidt 2024].

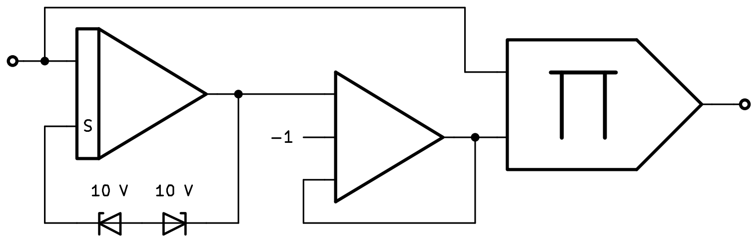

Figure 2: Simple voltage controlled unidirectional memristor model

over an input signal and use the integrator output to control a multiplier representing the "conductance".

Figure 2 shows the analog computer setup. The output signal of the integrator is limited to the interval $[-1, 1]$ by means of the two Z-diodes connected in series between its output and its summing junction.[4] Since a memristor cannot perform a sign-change, this output signal has to be mapped to the interval $[0, 1]$ which is done by the summer following the integrator. The direct feedback connection turns the summer into a summer with a common summing factor of $\frac{1}{2}$, thus adding -1 to the integrator output yields the desired output value interval at the output of the summer.

Figure 3 shows the characteristic curve of this simple model obtained by feeding its input with a sine signal and plotting this input signal (x axis) against its output value (y axis). The characteristic "pinched curve" is clearly visible.[5]

Using this model it is now possible to set up more complex circuits incorporating these novel circuit elements, albeit only as an "analog twin".

[4] Typically these two limiting diodes are not even necessary.

[5] The excitation sine signal was generated by two integrators and one inverter in a loop, solving $\ddot{y} = -y$ with one of the integrators having to amplitude limiting Z-diodes in series between its output and summing junction input.

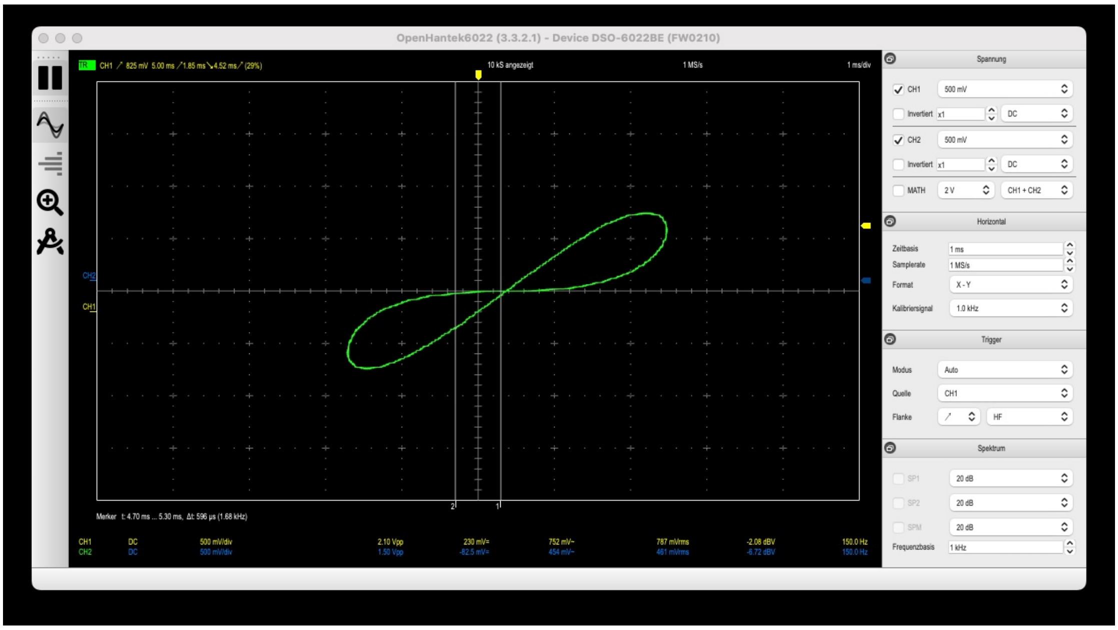

Figure 3: Behaviour of the simple memristor model

3 Is there a simpler way?

This circuit can be even further simplified by restricting the output range of the integrator to the interval $[-1, 0]$ instead of $[-1, 1]$ as shown in figure 4.[6] The Z-diode is optional as in the circuit above and guarantees that the output cannot exceed this interval.

This restricted interval requires an initial condition of $+\frac{1}{2}$ so that the integration starts at $-\frac{1}{2}$. Since its size is only half of the original interval, the input signal must also be scaled by $\frac{1}{2}$. Saving the summer thus comes at the extra cost of two additional coefficient potentiometers. The behaviour of this simplified setup is shown in figure 5.

[6]Without the implicit change of sign of the summer the interval must run from -1 to 0.

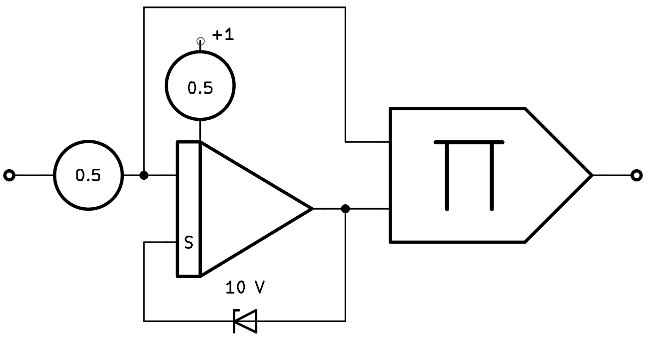

Figure 4: Simplified memristor model

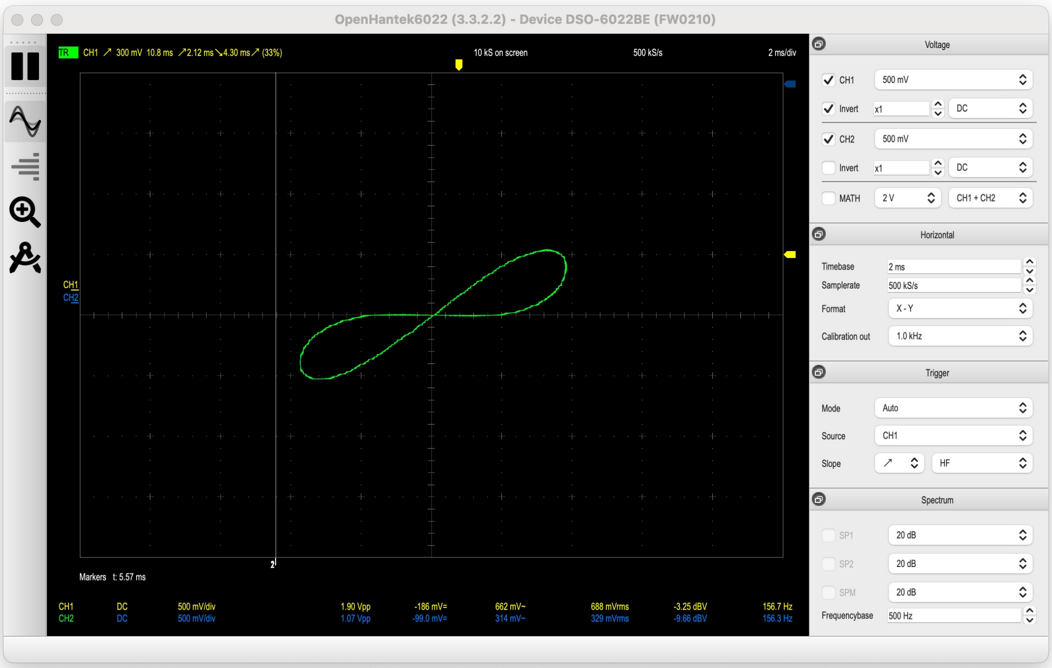

Figure 5: Behaviour of the simplified memristor model

Happy analog computing!

References

[Chua 1971] Leon O. Chua, "Memristor – The Missing Circuit Element", in *IEEE Transactions in Circuit Theory*, Vol, CT-18, No. 5, September 1971, pp. 507–519

[Schmidt 2024] Heidemarie Schmidt, "Prospects for memristors with hysteretic memristance as so-far missing core hardware element for transfer-less data computing and storage", in *Journal of Applied Physics*, 135, 200902, 2024