Analog Computer Applications
Application Note #11
Undergraduate Engineering Physics Space-Themed STEM Project

Hovering on Mars: Laser Power Beaming (A Simulation)

1 Introduction

At thirty meters above the surface of Mars, a one-kilogram, twin-rotorcraft helicopter Martian scout (eyes in the sky) hovers without batteries. Its rotor engine is powered by a solar panel that is energized from an infrared laser mounted on the planet's surface. For this simulation, the scout is restricted to vertical motion. Its mission is to use an onboard camera to seek out safe and interesting locations in which to direct a land rover (boots on the ground).

Initially, with the solar panel illuminated, the scout hovers at an altitude of thirty meters. At some point in time, the decision is made to it return the craft to the surface. This is accomplished by momentarily blocking the laser beam until the scout drops to an altitude of about twenty-seven meters with a speed of four meters per second (see page 4 for calculations).

When the scout reaches an altitude of twenty-seven meters, the beam is unblocked as the probe continues its descent with its solar panel fully energized. Note that a safe touch down speed is approximately one meter per second. The blocked-beam descent phase will be solved analytically to provide updated initial conditions.

It is assumed that the laser power delivered to the solar panel is not significantly attenuated as it propagates through the Martian atmosphere. Thus, the solar panel output will not change and power to the rotors will produce a constant thrust until the scout touches down, at which time the laser would be deactivated.

For this application note, quadratic drag is assumed.

For comparison, analog computer, numerical, and analytical methods will be employed with the results displayed in table 1.

2 Mathematical Modeling

Phase 1: Blocked-beam descent (analytical method only)

Starting with Newton's second law of motion, and assuming down to be positive (forward),

$$Ma = F_{net}$$

$$Ma = -drag + weight$$

$$Ma = -kv^2 + Mg$$

$$a = -k/M\, v^2 + g$$

$$dv/dt = -k/M\, v^2 + g$$

$$dv/dt = -k/M\, (v^2 - Mg/k) \qquad (1A)$$

Setting $dv/dt = 0$ and solving for terminal speed, v_T,

$$v_T^2 = Mg/k \qquad (2A)$$

Rewriting (1A),

$$dv/dt = -k/M\, (v^2 - v_T^2)$$

Using the chain rule for differentiation, $dv/dt = v\, dv/dz$. Thus,

$$v dv/dz = -k/M\, (v^2 - v_T^2)$$

where, z (assumed to be positive) is the distance traveled from its initial hovering position.

Continuing,

$$\mathrm{vdv}/(\mathrm{v}^2 - \mathrm{v}_\mathrm{T}^2) = -\mathrm{k}/\mathrm{M}\,\mathrm{dz}$$

$$\int_0^{\mathrm{v}} \mathrm{vdv}/(\mathrm{v}^2 - \mathrm{v}_\mathrm{T}^2) = -\mathrm{k}/\mathrm{M} \int_0^{\mathrm{z}} \mathrm{dz} \qquad (3\mathrm{A})$$

By inspection, the the right-hand side of (3A) becomes $-kz/M$.

Solving the integral on the left-hand side of (3A) requires a little more effort!

Setting $p = v^2 - v_T^2$,

$dp = 2vdv$, or $vdv = dp/2$, and $p(0) = 0^2 - v_T^2 = -v_T^2$

Substituting $dp/2$ for vdv,

$$1/2 \int_{p(0)}^{p} \frac{dp}{p} = 1/2 \ln\ |p/p(0)|$$

$$= 1/2\ \ln|(v^2 - v_T^2)/(-v_T^2)|$$

$$= 1/2\ \ln|1 - (v/v_T)^2| \qquad \text{since } v < v_T,$$

$$= 1/2\ \ln(1 - (v/v_T)^2)$$

Equating this to the right-hand side of (3A),

$$\ln(1 - (v/v_T)^2) = -2kz/M$$

Following a bit of algebra,

$$v = v_T\sqrt{(1 - e\hat{\ }(-2zk/M))}$$

Letting $k = 0.10$ kg/m, $M = 1.00$ kg, $g = 3.71$ m/s^2, and using (2A),

$$v_T = 6.09 \text{ m/s}$$

and

$$v = 6.09\ \text{m/s}\ \sqrt{}(1 - e\hat{}(-0.20\ \text{m}^{-1}\ z)$$

At $z = 2.82$ m, $v = 6.09\ \text{m/s}\ \sqrt{}(1 - e\hat{}(-0.20\ \text{m}^{-1} \times 2.82\ \text{m})) = 4.00$ m/s and

altitude = 30 m – 2.82 m = 27.18 m ≅ 27 m.

Phase 2: Unblocked-beam descent

Starting with Newton's second law of motion, and assuming down to be positive (forward),

$$Ma = F_{net}$$

$$Ma = -\text{drag} + \text{weight} - \text{thrust}$$

$$Ma = -kv^2 + Mg - T$$

$$a = -k/M\ v^2 + (Mg - T)/M$$

$$dv/dt = -k/M\ v^2 + (Mg - T)/M$$

$$dv/dt = -k/M\ (v^2 - (Mg - T)/k) \qquad (1B)$$

Setting $dv/dt = 0$ and solving for the terminal speed, v_T,

$$v_T^2 = (Mg - T)/k \qquad (2B)$$

Rewriting (1B),

$$dv/dt = -k/M\ (v^2 - v_T^2)$$

Using the chain rule from calculus,

$$v dv/dz = -k/M\ (v^2 - v_T^2)$$

$$v dv/(v^2 - v_T^2) = -k/M\ dz$$

For mathematical convenience, z will be set from 2.82 m to 0 m.

$$\int_{v(0)}^{v} vdv/(v^2 - v_T^2) = -k/M \int_{0}^{z} dz \qquad (3B)$$

Following the mathematical techniques used in phase 1,

the integral on the right becomes $-kz/M$ and

the integral on the left becomes

$1/2 \ln|(v^2 - v_T^2)/(v^2(0) - v_T^2)|$. However, this time $v(0) = 4.00$ m/s..

Since $v(0)$ and $v > v_T$,

$$1/2 \ln((v^2 - v_T^2)/(v^2(0) - v_T^2)) = -kz/M$$

Following a bit of algebra,

$$v = \sqrt{}((v^2(0) - v_T^2)e^\wedge(-2kz/M) + v_T^2)$$

Substituting numerical values,

$$v = \sqrt{}(15e^\wedge(-0.20 \text{ m}^{-1} z) + 1) \text{ m/s}$$

Requiring $v_T = 1.00$ m/s and using (2B),

thrust, $T = 1.00 \text{ kg} \times 3.71 \text{ m/s}^2 - 0.100 \text{ kg/m} \times (1.00 \text{ m/s})^2 = 3.61$ N.

This is the combined thrust for the rotors.

Substituting values for k, M, and v_T, and omitting units for clarity,

$$dv/dz = -0.100/1.00\ (v^2 - 1^2)/v$$

$$dv/dz = -0.100(v^2 - 1)/v$$

3a Computer setup (patch cord version)

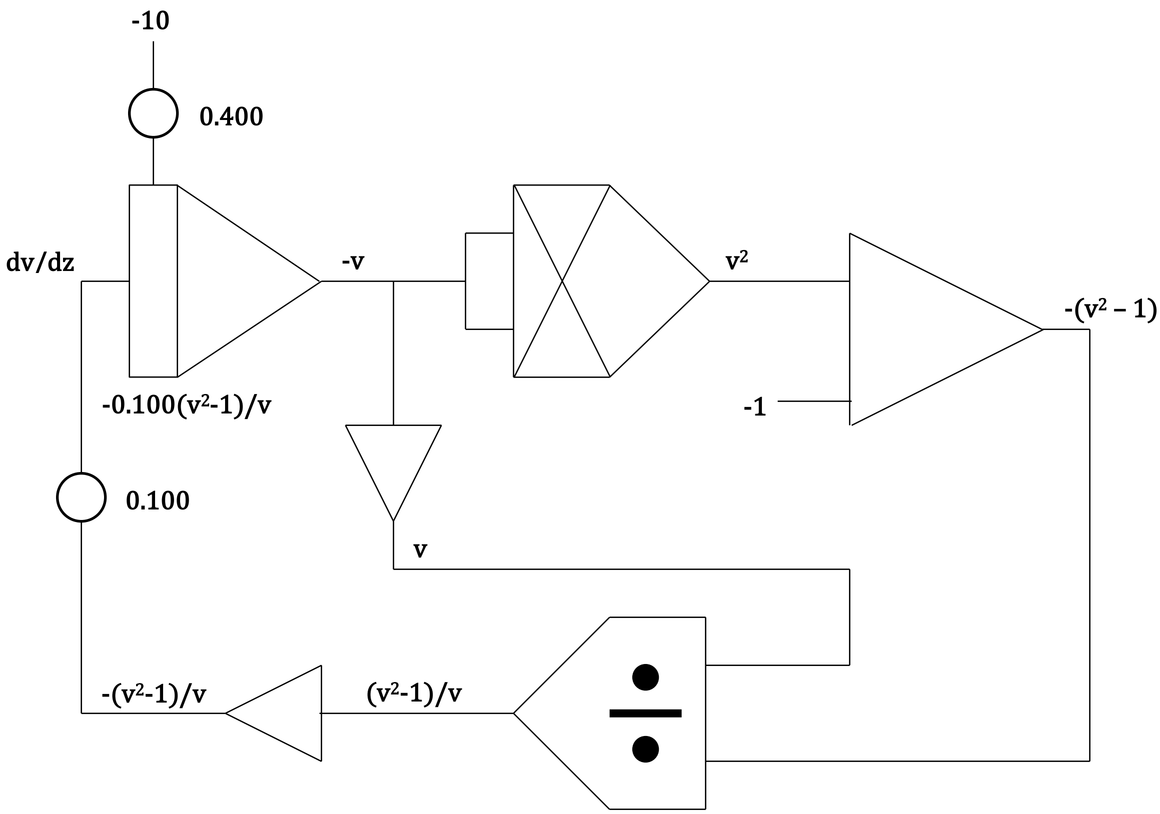

Figure 1: Computer setup

3b Computer setup (IC/discrete component version)

Arrow Color Code:

Red arrows connect to VCC.
Blue arrows connect to VEE.
Green arrows connect to ground.

Integrated Circuit Chip Code:

741 n = Op Amp n
633 m = Analog Multiplier m

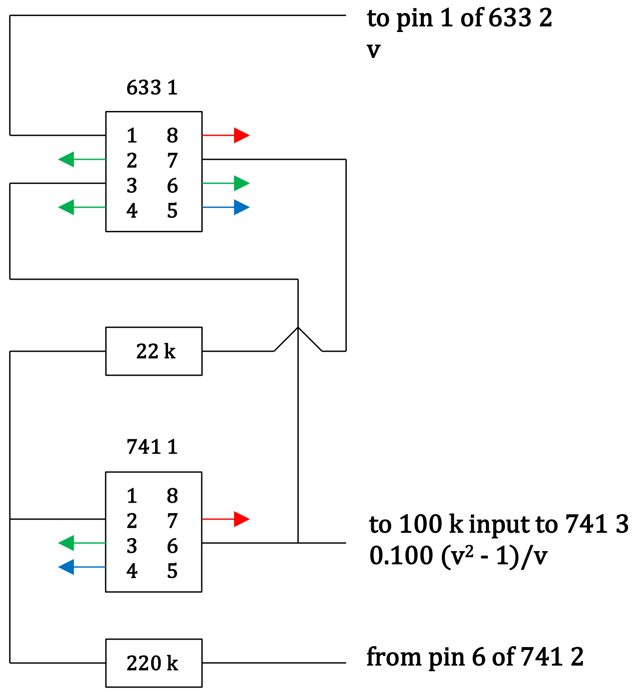

Figure 2: Basic breadboard layout for divider circuit

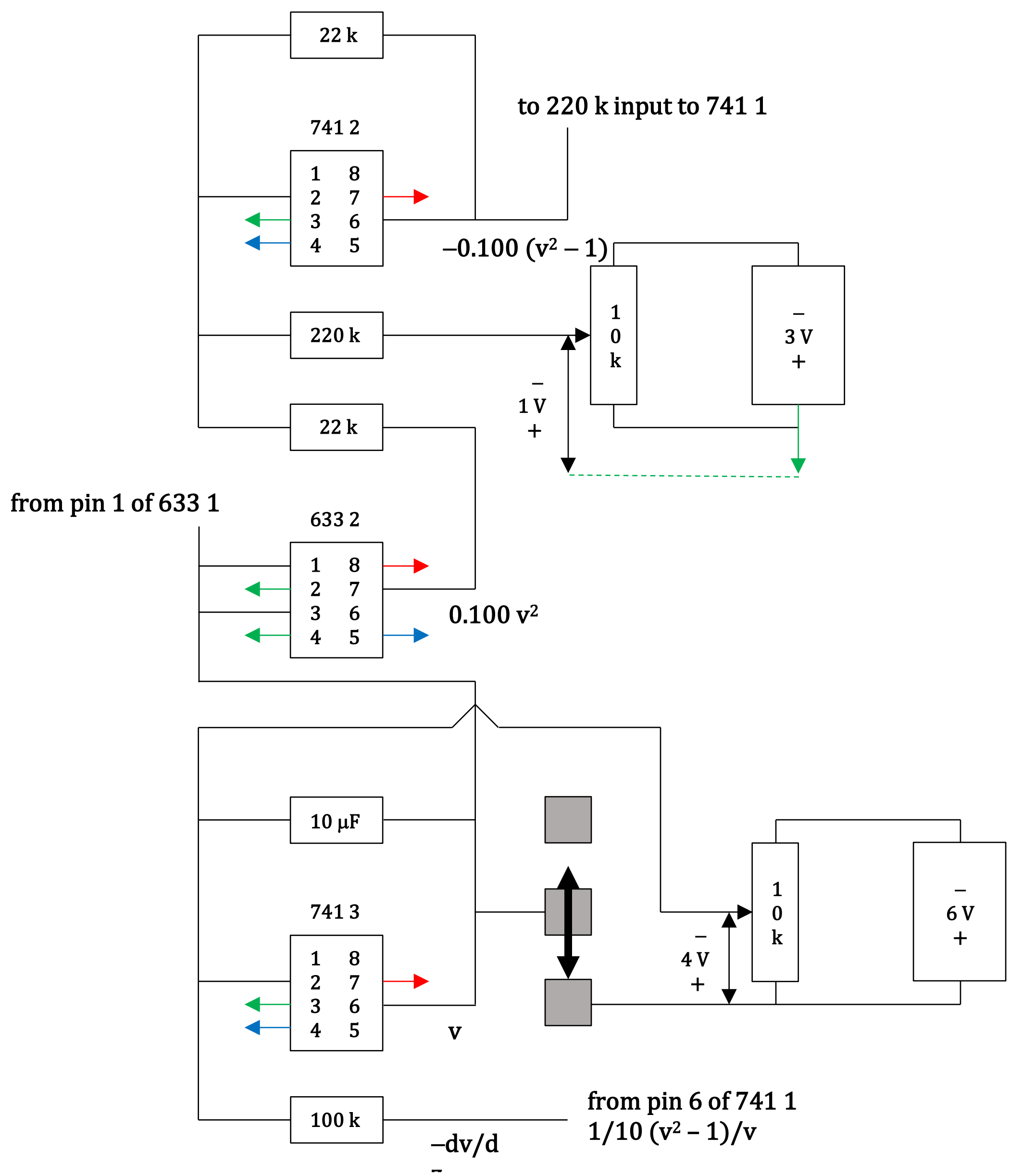

Figure 3: Basic breadboard layout for hovering on Mars

4 Numerical Method

(Euler's Predictor-Corrector method using a hand-held programmable calculator)

Code: TI-BASIC

```
PROGRAM:HOVERING
:ClrHome:ClrDraw
:"DV/DT=–0.1(V²–1)/V"
:"INITIAL CONDITION:"
:0→Z:4→V
:"STEP SIZE:"
:0.5→H
:Fix 2
:Lbl 1
:If Z>30:Then
:Goto 2:Else
:Disp {Z,V}
:–0.1(V²–1)/V→F
:Z+H→Z
:V+HF→W
:–0.1(W²–1)/V→S
:(F+S)/2→A
:V+HA→V
:Pause
:Goto1
:Lbl 2
:End
```

5 Results

z (m) from oscilloscope scale: 1 s = 1 m	Analog computer v (m/s) estimated from oscilloscope	Analytical v (m/s)	Euler's Predictor-Corrector Method v (m/s) Calculator
00.0	4.0	4.00	4.00
05.0	2.5	2.55	2.55
10.0	1.8	1.74	1.74
15.0	1.5	1.32	1.32
20.0	1.3	1.13	1.13
25.0	1.2	1.05	1.05
27.2 touch down	1.2	1.03	1.03

Table 1: Solution comparisons for unblocked-beam descent

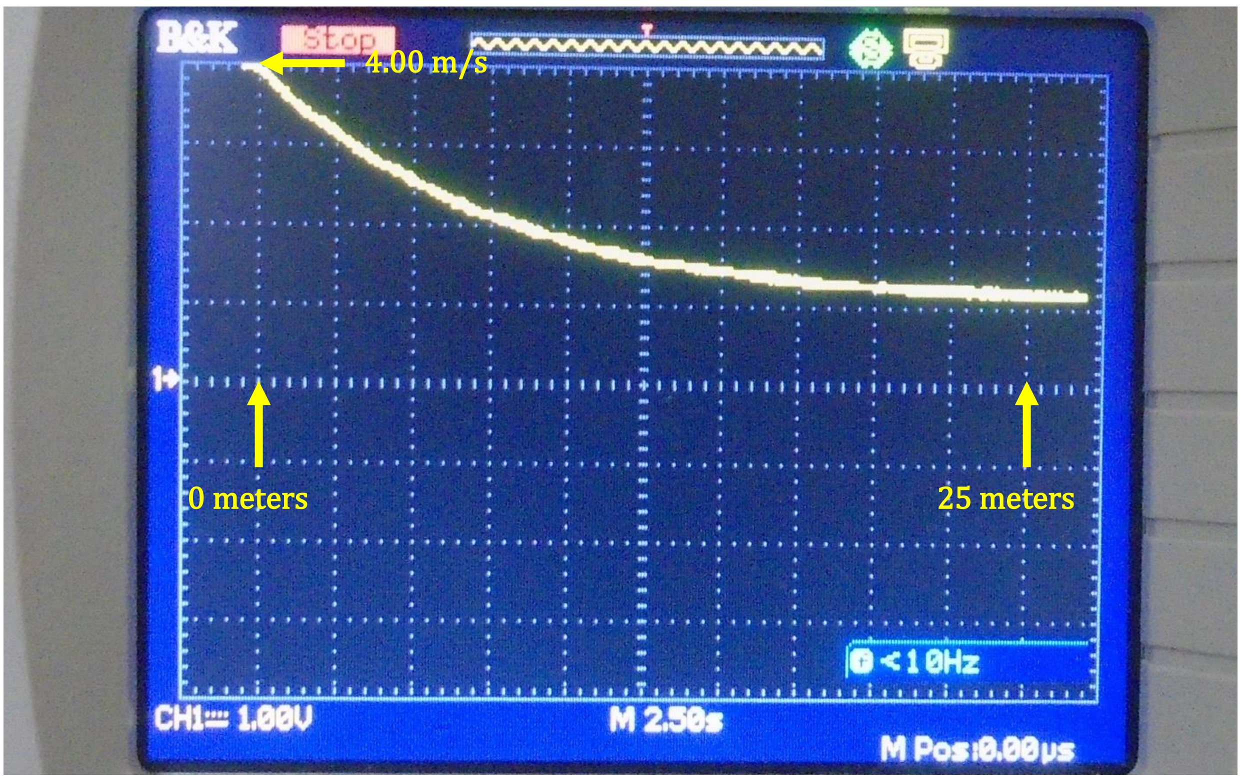

Figure 4: Martian scout descent speed during powered phase*

*For this application note, the oscilloscope display was produced during a single run using a differential equation analog computer constructed from operational amplifiers, analog multipliers, and discrete components with tolerances within 20%.

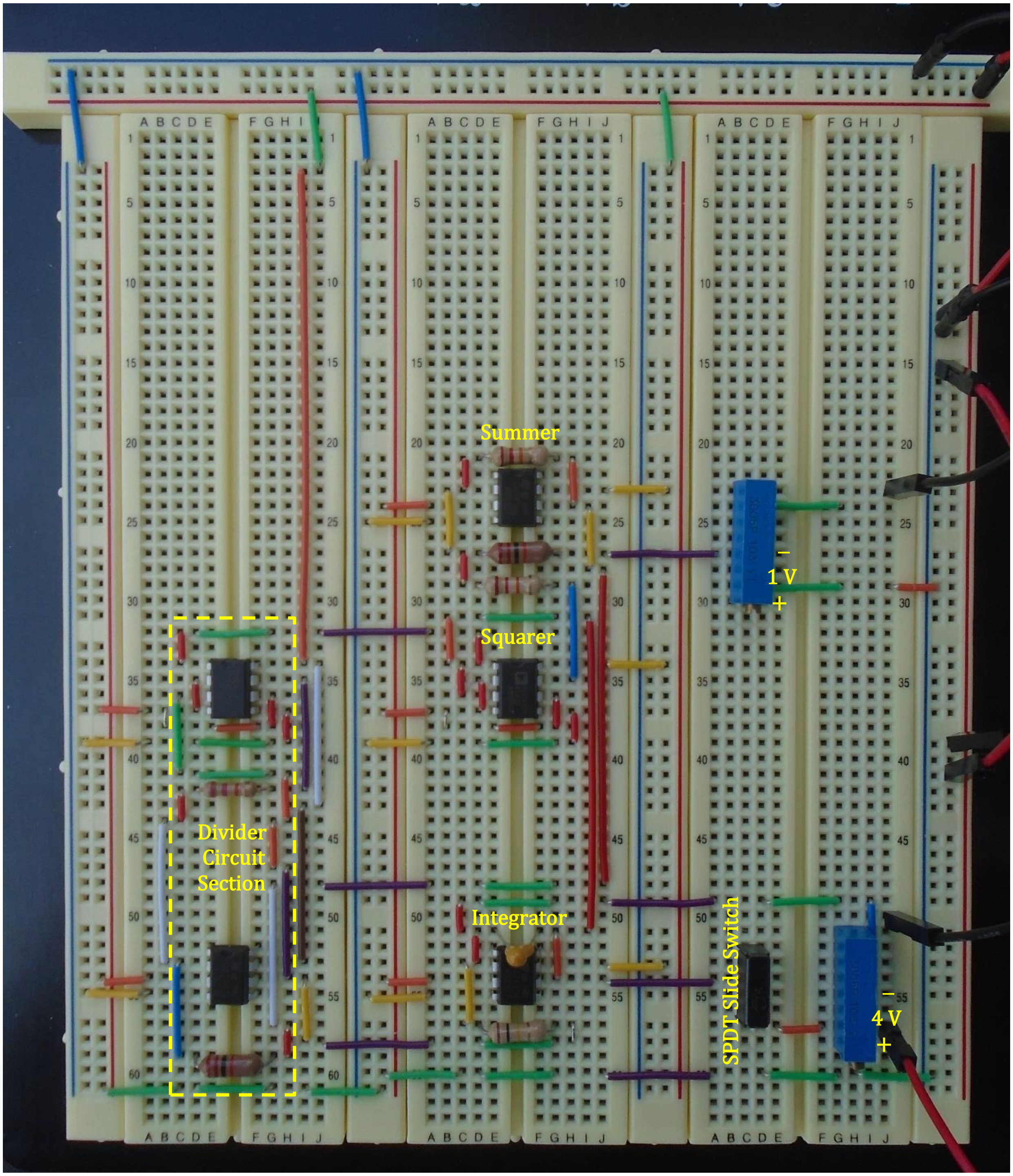

Figure 5: Basic breadboard layout